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Book Review: *Stability of Thermodynamic Systems*

Stability of Thermodynamic Systems. Edited by J. Casas-Vázquez and G. Lebon, Lecture Notes in Physics, No. 164. Springer-Verlag, Berlin, 1982. 321 pages.

This book constitutes the proceedings of a conference held in 1982 at the Bellaterra School of Thermodynamics, Autonomous University of Barcelona, whose intended purpose is to present both the recent progress in, and the general framework of, the field of nonequilibrium instabilities. The first half of the volume consists of three lectures which successfully introduce the uninitiated to the thermodynamic framework, mathematical methods, and basic phenomenology most often encountered in this field. However, a certain amount of effort is required to follow some of the arguments owing to the intrinsic difficulty of the subject. The second half of the book is devoted to nine lectures covering instabilities in a wide variety of fields, as we discuss below. The second half of the book is not, in my opinion, as successful as the first in introducing the nonexpert to the relevant instabilities in some fields. It is sometimes difficult to distinguish among comments that have a theoretical basis, an intuitive basis, or are just speculation.

The first lecture, by J. Casas-Vázquez, "Thermodynamic Theory of Stability," begins with a terse, but clear, review of the Gibbs theory of equilibrium thermodynamics. The question of stability and its relation to the second variation of the internal energy (entropy) is addressed, and the lack of a metric in the Gibbs space is discussed. The stability argument is geometric and relies on identifying the coefficients in the diagonal quadratic form of the second variation with second derivatives of Massier–Planck functions. The formal arguments are very much for the expert in the field; however, the examples make the formalism accessible to the nonexpert.

The stability of nonequilibrium systems in continuum mechanics and thermodynamics in the context of local equilibrium theory is also reviewed. In particular, Onsager's reciprocal relations and the principle of minimum entropy production are discussed. The concept of excess entropy production is used to define Lyapunov stability in the nonlinear range of irrevers-

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ible phenomena and the notion of dissipative structures is introduced. A brief excursion into the realm of generalized thermodynamics is made, wherein such limitations of standard theory as the infinite speed of temperature fluctuations in a medium are resolved.

The lecture "Mathematical Methods in Stability Theory," by G. Lebon, addresses the fundamental problem associated with the notion of the size of a perturbation and its eventual evolution, i.e., stability, from a number of mathematical viewpoints. Lyapunov's definition of stability in both discrete and continuous systems based on the norm of the separation between the perturbed and unperturbed solutions is reviewed.

The Bénard problem serves as an introduction to both linear and nonlinear stability theory and establishes preliminary values of the relevant parameters at which the linear solution bifurcates and also the sufficient conditions for instability. Nonlinear effects are drawn into the discussion through positive definite functionals of the perturbations which decrease in the course of time, e.g., the energy of the flow field. The contact between the mathematical analysis and thermodynamics is made through the identification of the excess entropy of Glansdorff and Prigogine as a Lyapunov function and then by using Lyapunov's theorem to establish stability of the reference state. This theory is unsatisfactory for fluid flow, necessitating a discussion of various perturbative methods for determining the stability of the flow field. The final strategy for addressing the question of stability is bifurcation theory for use when the details of the flow are not of interest.

It is pointed out by C. Pérez-Garcia in "Some Physical Mechanisms of Hydrodynamical Instabilities" that no general theory of far-from-equilibrium instabilities exists, i.e., no theory of nonequilibrium phase transitions, so a comparative study of instabilities in hydrodynamical systems is presented. The systems discussed are Rayleigh–Bénard convection, Taylor–Couette instability, and Poiseuille flow. A historical review of the first instability is presented, including the extension to thermodynamic convection. Many technical details are omitted and referred back to Lebon's lecture. The discussion then proceeds to liquid crystals in which the thermal convection in the nematic phase is addressed and the coupling between the temperature and molecular orientation fields is briefly considered. Just as linear stability analysis cannot predict the form of the cylindrical rolls or hexagonal patterns in the Rayleigh–Bénard problem, so too the Taylor vortices in the Couette flow cannot be predicted. Thus the direct effect of nonlinearities on stability properties is discussed using perturbation theory. The discussion here focuses on physical systems and experimental observations and thereby compliments the lecture of Lebon and extends beyond the organized dissipative structures into the chaotic regime of turbulent fluid flows. The conclusion drawn is that a general thermodynamic formalism for systems far from equilibrium is necessary in order to properly interpret existing data.

The lecture by D. Jou on "Hydrodynamic Fluctuations Near the Rayleigh–Bénard Instability" reviews the thermodynamic viewpoint intending to strengthen the analogy between second-order equilibrium phase transitions and nonequilibrium instabilities in hydrodynamics. He includes a discussion of critical exponents and introduces a phenomenological potential. This potential is related to the probability of a fluctuation of a given magnitude occurring and to the stability of the steady states of the hydrodynamic system. The effects of nonlinear terms on convective instability are explored. Unfortunately, the lecture focuses on the fluctuations in the velocity amplitude and not the phase.

P. Bergé in "Some Topics About the Transition to Turbulence" gives a terse presentation of a number of facts known about dynamic systems, discrete mappings, and chaos. Speculations on the relation of these topics to the transition to turbulence in a real fluid are made.

M. Dubois discusses the "Experimental Aspects of the Transition to Turbulence in Rayleigh–Bénard Convection." The transition in time from a stationary to a turbulent state is examined for a high Prandtl number fluid in a small box submitted to Rayleigh–Bénard convection. The stability of the spatial structures, i.e., rolls, is examined as the Rayleigh number is varied. A generic transition between two distinct spatial orderings with a transitory chaotic state intermediate in time, which may last from a few hours to a few days, is observed at a fixed R_a. The time-dependent properties of different structures, such as turbulent intermittences and biperiodic temporal turbulence, are examined. In particular, the phenomenon of dynamical phase locking is observed in the intermediate state.

In the lecture "Heat Flux in Convective Instabilities" M. Zamora discusses the energy balance in convective flow to determine the dependence of the velocity on Rayleigh number, $v \sim (R_a - R_{ac})^{1/2}$ for convective instability. The Nusselt number $N_u = 1 + (R_a - R_{ac})/R_{ac}$ shows that the heat flux is discontinuous at the critical point. Experimental support of this result is given and discussed.

The hydrodynamic instabilities in fluids with nondilute suspensions are modified such that control by nonlinearities could promote the existence of steady states far from equilibrium, as discussed by D. Ouemada in "Unstable Flows of Concentrated Suspensions." The viscosity of non-Newtonian fluids is discussed, e.g., plastic and pseudoplastic fluids, and are experimentally shown to be discontinuous, e.g., a suspension of monodispersive polymeric resins. The suggested mechanism is flow-induced modification in the concentration of the suspension. Some systematic speculations on flow instabilities in this model are presented.

In "Dissipative Structures and Oscillations in Reaction Diffusion Models with and Without Time-Delay," M. G. Velarde describes dissipative structures by means of worked out examples from biology, ecology, biochemistry, and semiconductor physics. A simplified model to mimic the sustained oscillations of continuous cultures of microorganisms is studied using Monte Carlo techniques. A semiclassical model of exciton generation and decay is investigated and an experiment to observe oscillating light emission due to oscillating exciton concentration is proposed. The steady states for a system of reaction–diffusion equations are studied, as is a model problem in which diffusion, advection, and time delay compete in an ecosystem.

A phenomenological equation consisting of a systematic part and a fluctuating part, the latter satisfying a fluctuation–dissipation relation, is discussed as a general model of continuum systems by J. M. Rubi in "Fluctuations in Electromagnetic Systems." A Langevin field equation for a ballast resistor is considered in some detail, as are the fluctuations near the instability of the electrodiffusive (Gunn) effect. The nonlinear fluctuations and associated dissipative currents in charged fluids are examined in this context and the equilibrium solution to a Fokker–Planck equation is derived in a discrete space.

P. Morgalef gives a qualitative discussion of "Instabilities in Ecology." The instabilities in a class of ecosystems are presented, e.g., bacterium in a cultured flask, evolution of a forest, and phytoplankton, as well as the process of sedimentation. One model for such systems rests on the thermodynamic concept of energy balance leading to nonequilibrium steady states.

C. Perellá in his lecture on "Strange Attractors" discusses the evolution of a dynamical system from an initial to a final state. In particular, examples of systems with trajectories showing a complicated limit behavior (strange attractor) are briefly presented, including the Lorenz and Rikitake attractors. Passing reference is made to the experiments suggesting a connection between strange attractors and hydrodynamic turbulence.

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